

Sub-Poissonian phononic population in a nanoelectromechanical system

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Population of a phononic mode coupled to a single-electron transistor in the sequential tunneling regime is discussed for the experimentally realistic case of intermediate electron-phonon coupling. Features like a sub-Poissonian bosonic distribution are found in regimes where electron transport drives the oscillator strongly out of equilibrium with only few phonon states selectively populated. The electron Fano factor is compared to fluctuations in the phonon distribution, showing that all possible combinations of sub- and super-Poissonian character can be realized.

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Introduction.— Condensed-matter physics and optics have often exchanged concepts and ideas, based on the common underlying structure of wave phenomena. These are essentially based on interference effects which, in the case of light, can be derived from classical wave equations. However, features exist which cannot be explained within a classical treatment, as e.g. squeezed states [1] and photon antibunching in resonance fluorescence [2]. On the other hand, in mesoscopic physics the focus is often on electron transport. The latter is naturally characterized in terms of current, and much information (e.g. carriers charge, process statistics, correlation effects) can be extracted from noise [3]. Of particular interest are then condensed-matter systems in which fermionic and bosonic degrees of freedom are coupled, and where electron transport induces the emission of nonclassical radiation. For example, it is known that electronic shot noise in a quantum point contact may be source of antibunched photons [4], and emission of antibunched phonons from a two-level quantum dot is expected when transport is characterized by bunching of tunneling electrons [5].

Nanodevices where mechanical motion is coupled to electric transport constitute in this sense a perfect subject for study [6]. Realizations of these nanoelectromechanical systems (NEMS) have been obtained e.g. with single oscillating molecules [7], semiconductor beams [8] and suspended carbon nanotubes [9]. NEMS are interesting dynamical systems and are expected to show many peculiar transport features ranging from shuttling instability [10] to avalanche-like transport [11]. Recently, attention has also been focused on the *mechanical* properties of NEMS, as it appears now that experiments are close enough to the quantum limit [12] to test theoretically predicted quantum features in the vibrational motion [13]. Of particular interest are those associated with the discrete energy states of the oscillator and, indeed, several proposals have been put forward to measure discrete number states [14]. Furthermore, it is well known that the distribution of oscillation quanta (phonons) in NEMS is strongly affected by the transport of electrons [15, 16], and even the existence of non-

classical number states induced by tunneling has been predicted [17].

In this Letter, we address the behavior of a harmonic oscillator coupled to a quantum dot, focusing on the distribution of unequilibrated phonons induced by electric transport. We show that it is possible to achieve a *selective* population of few phonon states such that the distribution of the phonon number l displays a sub-Poissonian behavior, i.e. $\text{Var } l < \langle l \rangle$. At the same time, we consider the zero-frequency current noise and show that the fluctuations of both the phonon distribution and the electron current can be enhanced or reduced with respect to Poissonian statistics one independently of the other.

Model & methods.—The system we consider is a gated single-electron transistor (SET) coupled to leads and to a harmonic oscillator. The SET Hamiltonian is described within the standard constant-interaction model for spinless particles. In particular, the charging-energy term is $H_c = E_c(n - n_g)^2$, where $(n - n_g)$ is the effective number of electrons on the SET and n_g is proportional to the charge induced by the gate. The oscillator and coupling terms are ($\hbar = 1$)

$$H_{\text{ph}} = \omega_0 b^\dagger b + \sqrt{\lambda} \omega_0 (b^\dagger + b) (n - n_g), \quad (1)$$

where b^\dagger creates vibrational excitations of energy ω_0 . The dimensionless parameter $\sqrt{\lambda}$ defines the strength of electromechanical interaction between the position of the oscillator and the effective charge on the SET. Such a term can be induced by an oscillating gate capacitively coupled to the dot [8]. The leads are Fermi liquids with $H_l = \sum_{k,\alpha=L,R} \epsilon_{k,\alpha} c_{k,\alpha}^\dagger c_{k,\alpha}$ and chemical potentials $\mu_{L,R} = \mu_0 \pm eV/2$, where V is the bias voltage. In the limit $\omega_0, eV, k_B T \ll E_c, \Delta E$, where ΔE is the average single-particle level spacing, the SET excess occupancy is limited to 0, 1 and we can focus on the lowest unoccupied single-particle level ξ . The total Hamiltonian can then be written as $H = \epsilon n + H_{\text{ph}} + H_l + H_t$, where $\epsilon = \xi + 2E_c(1/2 - n_g)$ and $n = d^\dagger d$ are respectively the energy and the occupation number of the single level, and $H_t = \sum_{k,\alpha=L,R} (t_\alpha c_{k,\alpha}^\dagger d + \text{h.c.})$. Here, t_α are the tunneling amplitudes, with asymmetry $A = |t_R|^2/|t_L|^2$.

Being interested in the weak-tunneling limit, we treat H_t as a perturbation. It is then convenient to perform a canonical transformation to make the unperturbed Hamiltonian diagonal in the system variables n, l . The desired transformation is the Lang-Firsov polaron transformation $\bar{O} = UOU^\dagger$ with $U = \exp \eta(b - b^\dagger)$ and $\eta = \sqrt{\lambda}(n - n_g)$ [15]. The transformed Hamiltonian is given by $\bar{H} = \bar{\epsilon}n + \omega_0 b^\dagger b + H_l + \bar{H}_t$, where $\bar{\epsilon} = \epsilon - \lambda\omega_0$ and

$$\bar{H}_t = \sum_{k, \alpha=L,R} (t_{\alpha} c_{k, \alpha}^\dagger d e^{-\sqrt{\lambda}(b^\dagger - b)} + h.c.). \quad (2)$$

In the polaron picture, coherences between states with different phonon number can be neglected as far as the level broadening γ induced by tunneling is the smallest energy scale into play [15], i.e. $\gamma \ll \omega_0, k_B T$. In this limit, the reduced density matrix $\bar{\rho}$ of the SET+oscillator system in the polaron picture is diagonal both in n and l , and the dynamics is well described by the rate equations $\partial_t \bar{P}_{0(1), l} = \sum_{l'} [\Gamma_{o(i)}^{l'l} \bar{P}_{1(0), l'} - \Gamma_{i(o)}^{ll'} \bar{P}_{0(1), l}]$ for the populations $\bar{P}_{n, l} = \langle n, l | \bar{\rho} | n, l \rangle$. Here, $\Gamma_{i(o)}^{ll'} = \sum_{\alpha} \Gamma_{\alpha i(o)}^{ll'}$ are the total rates for tunneling in (out of) the level, and

$$\begin{aligned} \Gamma_{\alpha i}^{ll'} &= 2\pi\nu |t_{\alpha}|^2 X^{ll'} f_{\alpha}(\omega_0(l' - l)) \\ \Gamma_{\alpha o}^{ll'} &= 2\pi\nu |t_{\alpha}|^2 X^{ll'} [1 - f_{\alpha}(\omega_0(l - l'))], \end{aligned} \quad (3)$$

where $f_{\alpha}(x) = f(x + \bar{\epsilon} - \mu_{\alpha})$ and $f(x)$ is the Fermi function. The coefficients $X^{ll'} = |\langle l' | e^{-\sqrt{\lambda}(b^\dagger - b)} | l \rangle|^2$ are the Franck-Condon (FC) factors [11, 15] and ν is the density of states of the leads. In the following we assume $\mu_0 = \xi - \lambda\omega_0$ so that $n_g = 1/2$ defines on-resonance conditions. We focus on the regimes of weak ($\lambda \ll 1$) and intermediate ($\lambda \approx 1$) phonon coupling, where cotunneling is negligible out of the Coulomb-blockaded regions [11].

Electronic and phononic expectation values can be evaluated in the polaron picture as $\langle O \rangle = \text{Tr}[\bar{O}\bar{\rho}] = \sum_{n,l} \langle n, l | \bar{O} | n, l \rangle \bar{P}_{n, l}$. It is useful to define also a “hybrid” average $\langle O \rangle_{\bar{\rho}} = \text{Tr}[\bar{O}\bar{\rho}]$. In terms of $\langle \cdot \rangle_{\bar{\rho}}$, we can write

$$\langle l \rangle = \langle l \rangle_{\bar{\rho}} + \langle \eta^2 \rangle_{\bar{\rho}}, \quad (4)$$

$$\langle l^2 \rangle = \langle l^2 \rangle_{\bar{\rho}} + 4\langle \eta^2 l \rangle_{\bar{\rho}} + \langle \eta^2 \rangle_{\bar{\rho}} + \langle \eta^4 \rangle_{\bar{\rho}}, \quad (5)$$

where we have used the fact that $\bar{b} = b - \eta$ and that $\bar{\rho}$ is diagonal in the considered weak-tunneling limit. Note that for operators like n , which are unchanged by the canonical transformation, it is $\langle n \rangle_{\bar{\rho}} = \langle n \rangle$.

From the stationary solution of the master equation, the phonon Fano factor $F_{\text{ph}} = \text{Var} l / \langle l \rangle$ can be directly calculated in terms of Eqs.(4) and (5). The electronic Fano factor $F = S/2e\langle I \rangle$ is evaluated following Ref. 18.

Results.— We first consider the phonon Fano factor F_{ph} . Our main result is that, in the presence of asymmetry of the tunneling barriers and specific voltage conditions, the phonon distribution shows a sub-Poissonian behavior $F_{\text{ph}} < 1$ (see Fig. 1a).

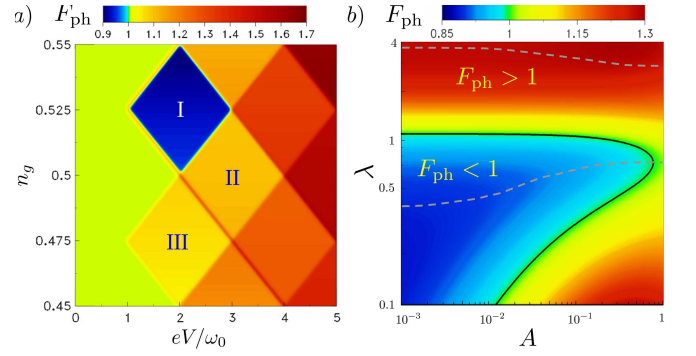


FIG. 1: a) Phonon Fano factor F_{ph} as a function of voltage V and n_g , at $A = 0.1$ and $\lambda = 0.7$. b) Density plot of F_{ph} as a function of A and λ for $eV = 2\omega_0$, $n_g = 0.525$ (middle of region I in a). Black line: contour $F_{\text{ph}} = 1$. The region between the dashed lines encloses the four-state regime (see text). In both panels: $k_B T = 0.01 \omega_0$, $E_c = 10 \omega_0$. Color scales on the top.

In particular, for $A < 1$ ($A > 1$) the most favorable region of the $V - n_g$ plane for having $F_{\text{ph}} < 1$ is region I (region III). A sub-Poissonian F_{ph} can also be obtained in region II, but only in the limit of very strong asymmetry (not shown). For definiteness, in the following we assume $A < 1$ and we focus mainly on region I which, in general, is limited by the following conditions: $\omega_0 \leq eV \leq 3\omega_0$ and $1/2 \leq n_g \leq 1/2 + \omega_0/2E_c$.

Here, the phonon Fano factor shows a crossover between sub- and super-Poissonian behavior as a function of A and λ (see Fig. 1b). As a rule of thumb, a sub-Poissonian F_{ph} requires $\lambda \lesssim 1$ and it is favored by strong asymmetries. Interestingly, for intermediate values of the electron-phonon coupling $\lambda \approx 1$, it is $F_{\text{ph}} < 1$ already for asymmetries which are experimentally feasible, $A \lesssim 1$.

Theoretically, the super- (sub-) Poissonian character of F_{ph} is more easily studied in terms of the parameter $Q = \text{Var} l - \langle l \rangle$, being $F_{\text{ph}} < 1$ only if $Q < 0$. Let us write $Q = Q_{\bar{\rho}} + \Delta Q$, where $Q_{\bar{\rho}} = \langle l^2 \rangle_{\bar{\rho}} - \langle l \rangle_{\bar{\rho}}^2 - \langle l \rangle_{\bar{\rho}}$ and

$$\begin{aligned} \Delta Q &= 2\lambda \langle l \rangle_{\bar{\rho}} [n_g^2 - (2n_g - 1)(2 - \langle n \rangle_{\bar{\rho}})] \\ &+ 4\lambda(2n_g - 1) \langle l(1 - n) \rangle_{\bar{\rho}} + \lambda^2(2n_g - 1)^2 \text{Var} n. \end{aligned} \quad (6)$$

Taking into account that $n \in \{0, 1\}$, it is easy to show that it is always $\Delta Q > 0$ in region I. Therefore, a sub-Poissonian F_{ph} requires necessarily $Q_{\bar{\rho}} < 0$.

The quantity $Q_{\bar{\rho}}$ can be evaluated in terms of the phonon distribution in the polaron frame $\bar{P}_l = \bar{P}_{0, l} + \bar{P}_{1, l}$. Note that if \bar{P}_l obeyed a thermal distribution $\bar{P}_l = e^{-\beta l \omega_0} (1 - e^{-\beta \omega_0})$, it would be $Q_{\bar{\rho}} = (e^{\beta \omega_0} - 1)^{-2} \geq 0$, while if \bar{P}_l follows the Poisson distribution it would obviously be $Q_{\bar{\rho}} = 0$. On the other hand, when only the two lowest vibrational levels are occupied, $\bar{P}_0 + \bar{P}_1 = 1$, it is $Q_{\bar{\rho}} = -\langle l \rangle_{\bar{\rho}}^2 \leq 0$.

A phonon distribution having only the first few states occupied is therefore a promising candidate for observing

$F_{\text{ph}} < 1$. Indeed, our numerical investigations strongly suggest that a sub-Poissonian phonon Fano factor requires a slender phonon distribution: in fact $F_{\text{ph}} < 1$ is solely observed at low voltages, in the presence of asymmetry and preferably off-resonance, which are all conditions which favor a narrow $\bar{\mathcal{P}}_l$ [15].

Remarkably, for $\lambda = 1$ the exact stationary solution of the rate equation satisfies the condition $\bar{\mathcal{P}}_0 + \bar{\mathcal{P}}_1 = 1$ always in region I. This fact can be understood observing that in region I the only energetically allowed transitions which increase the phonon number are $(0, l) \rightarrow (1, l+1)$, see Eq.(3). As a consequence, at low temperatures $k_B T \ll \omega_0$, excited phonons states can only be populated via a series of subsequent tunneling events such as $(0, 0) \rightarrow (1, 1) \rightarrow (0, 1) \rightarrow (1, 2)$. However, for $\lambda = 1$ the transition $(1, 1) \rightarrow (0, 1)$ is forbidden because the FC factor $X^{11} = e^{-\lambda(1-\lambda)^2}$ vanishes. In this case, the dynamics of the system is frozen to the states with $l \leq 1$ and it can be solved exactly by considering the reduced four-state model represented in Fig. 2a. Within this model, we obtain

$$F_{\text{ph}}|_{\lambda=1} = 1 + \frac{3 - 8n_g + 4n_g^2 + 2A(3 - 8n_g + 5n_g^2)}{(2 + A)[n_g^2(A + 2) - 4n_g + 3]},$$

which gives e.g. $F_{\text{ph}} = 0.96$ for $n_g = 0.54$ and $A = 0.1$.

The four-state model of Fig. 2a is often a good approximation of the full numerical solution also for $\lambda \neq 1$. Within this model, we derive an analytical expression for F_{ph} which, however, is too long to be reported here. This approximates the exact numerical result with an error smaller than 1% in all region enclosed between the dashed lines in Fig. 1b, and therefore it allows us to investigate analytically the crossover between super- and sub-Poissonian phonon Fano factor. In particular, we find that it can be $F_{\text{ph}} < 1$ only for values of the electron-phonon coupling smaller than a certain critical value λ_{cr} which, up to order $(n_g - 1/2)^2$, is given by

$$\lambda_{\text{cr}} = \frac{1}{4(n_g - 1)^2 + 2A(n_g^2 - 4n_g + 2)}. \quad (7)$$

This equation describes very accurately the upper part of the contour line $F_{\text{ph}} = 1$ in the phase diagram Fig. 1b, deviating from the exact result only for $A \rightarrow 1$.

The existence of a maximum critical value $\lambda_{\text{cr}} < 1$ can be understood qualitatively considering the limit of strong asymmetry $A \ll 1$. In this case, it is $\langle n \rangle \approx 1$ and $\text{Var } n \approx 0$, and from Eq.(6) one obtains directly $Q \approx \langle l \rangle_{\bar{\rho}} [2\lambda(n_g - 1)^2 - \langle l \rangle_{\bar{\rho}}]$, where we have used $Q_{\bar{\rho}} = -\langle l \rangle_{\bar{\rho}}^2$ in the four-state model. A strong electron-phonon coupling is thus unfavorable for $Q < 0$ in two respects: on one hand, it increases the weight of the positive term $\propto (n_g - 1)^2$; on the other one, it is well known that $\bar{\mathcal{P}}_{l=0} \rightarrow 1$ as λ is increased [15], so that $\langle l \rangle_{\bar{\rho}} \rightarrow 0$.

We can conclude that the sub-Poissonian Fano factor is induced by a phonon distribution $\bar{\mathcal{P}}_l$ in which only

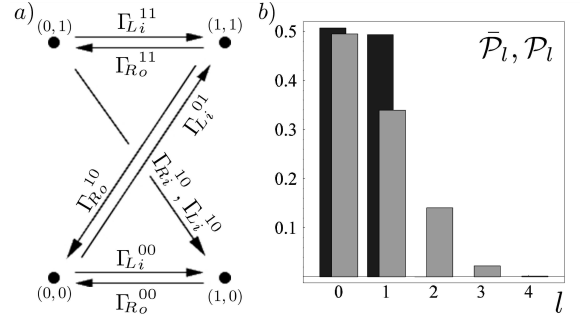


FIG. 2: a) Set of states included in the four-state model. States are labeled as (n, l) , arrows represent the relevant transitions in region I for $k_B T \ll \omega_0$. b) $\bar{\mathcal{P}}_l$ (black) vs. \mathcal{P}_l (gray) for $eV = 2\omega_0$, $n_g = 0.525$, $k_B T = 0.01\omega_0$, $E_c = 10\omega_0$ and $\lambda = 0.9$, $A = 0.01$.

the first few phonon states are populated and yet the occupation probability of the excited states is comparable with the one of the ground state. We refer to such a situation as a *selective population* of the phonon states.

Finally, we remind that $\bar{\mathcal{P}}_l$ is the phonon distribution in the polaron picture. The intrinsic phonon distribution is $\mathcal{P}_l = \sum_n P_{n,l}$ where $P_{n,l} = \langle n, l | \rho | n, l \rangle$ and $\rho = U^\dagger \bar{\rho} U$ is the density matrix in the original picture. Note that, in terms of \mathcal{P}_l , the average phonon number reads $\langle l \rangle = \sum_l l \mathcal{P}_l$, and similarly $\langle l^2 \rangle = \sum_l l^2 \mathcal{P}_l$. We can evaluate \mathcal{P}_l taking into account that $P_{n,l} = \sum_{l'} X_n^{ll'} \bar{P}_{n,l'}$, where $X_n^{ll'} = |\langle n, l | U | n, l' \rangle|^2$ are generalized FC-factors. Such a relationship is a consequence of $\bar{\rho}$ being diagonal in the weak-tunneling limit. Comparing \mathcal{P}_l and $\bar{\mathcal{P}}_l$, it is clear that one can speak of selective population *only* in the polaron picture (see Fig. 2b). However, what is important is that, in the presence of a selective population of $\bar{\mathcal{P}}_l$, the intrinsic phonon distribution \mathcal{P}_l shows a sub-Poissonian behavior, signaled by $F_{\text{ph}} < 1$.

Up to now, we have considered solely the characteristics of the phonon distribution induced by tunneling. However, it is well known that the transport properties of the system are in turn strongly affected by phonons. Signatures of this interplay are especially visible in the current Fano factor, which is very sensitive to the electron-phonon interaction [11, 19]. For example, a giant enhancement of F has been predicted as fingerprint of strong electron-phonon coupling [11]. Here, we consider intermediate coupling and we focus on the study of the (sub-) super-Poissonian character of F with respect to the one of F_{ph} . Interestingly, in region I all the possible combinations of F , $F_{\text{ph}} \leq 1$ can be obtained by tuning the asymmetry A and λ (see Fig. 3a). This is possible because the super- and sub-Poissonian character of F and F_{ph} have different physical origins. In fact, while $F_{\text{ph}} < 1$ presumes a selective population of phonon states, $F > 1$ is induced by a bunching of tunneling events [20].

A simple explanation of this mechanism can be given

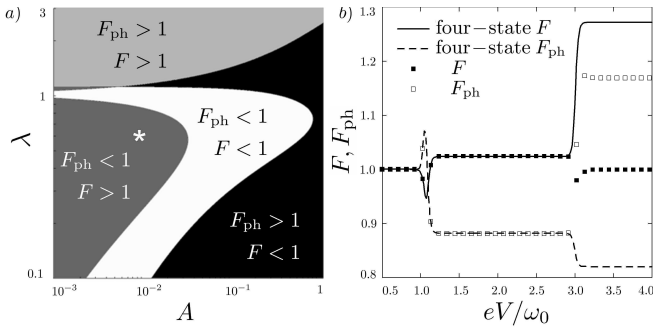


FIG. 3: *a)* Phase diagram of the possible combinations of F, F_{ph} in region I ($eV = 2\omega_0, n_g = 0.525$) depending on λ and A (see text for discussion). *b)* Plots of F_{ph} and F as a function of V for $n_g = 0.525$ and $\lambda = 0.6$, $A = 0.01$ corresponding to the asterisk in panel *a* for $eV = 2\omega_0$. Boxes: exact numerical solutions; lines: four-state approximation. In both panels $k_B T = 0.01 \omega_0$, $E_c = 10 \omega_0$.

in terms of the four-state model of Fig. 2*a*. For $\lambda \approx 1$, the electronic Fano factor can be written as:

$$F = \frac{4 + A^2}{(2 + A)^2} + \frac{4 + 2A + 14A^2 + 9A^3 - A^4}{(1 + A)^2(2 + A)^3}(1 - \lambda)^2.$$

For $\lambda = 1$ the system behaves as a spin degenerate single level so that it is always $F \leq 1$ [21]. For $\lambda \approx 1$, the transitions $(0,0) \leftrightarrow (1,1)$ and $(0,0) \leftrightarrow (1,0)$ act as two competing transport channels, whose relative weight is determined by the ratio $X^{01}/X^{00} = \lambda$. It follows that for $\lambda < 1$ the state $(1,1)$ is a trap state and blocks the transport through the other more conducting channel $(0,0) \leftrightarrow (1,0)$. In the presence of asymmetry, such a dynamical channel blockade [20] leads to bunching of tunneling events and to super-Poissonian current noise $F > 1$. However, as the difference between the two competing transport channels is fairly weak, F is only slightly above 1 (see Fig. 3*b*). The same mechanism occurs for $\lambda > 1$ but, in this case, it is transport through the excited state that is blocked by the occupation of $(1,0)$.

It is then clear why super-Poissonian current noise and sub-Poissonian phonon distribution can occur simultaneously only for $\lambda < 1$ when the trapping mechanism responsible for $F > 1$ also favors the selective population of the phonon states. Viceversa, for $\lambda > 1$ the occupation of the vibrational ground state is strongly favored since $(1,0)$ is the trap state, and the phonon distribution is mainly super-Poissonian. Note that outside region I the four-state model differs considerably from the exact results, which exhibit $F \leq 1$ and $F_{\text{ph}} > 1$ as expected from a fermionic and bosonic system, respectively.

Finally, a comment is in order. A suppressed phonon Fano factor $F_{\text{ph}} < 1$ has recently been predicted for an oscillator driven by a superconducting SET in the limit $\gamma \sim \omega_0$, and this has been interpreted as signature of a number-squeezed state [17]. In our case, instead, we obtain a sub-Poissonian distribution *without* squeezing.

This is ultimately a consequence of the loss of phase information in the weak-tunneling limit $\gamma \ll \omega_0$ [22].

In conclusion, we have shown that a sub-Poissonian phonon distribution can be achieved in a nanoelectromechanical system when tunneling induces a selective population of few phonon states. In addition, we have considered the electronic noise and we have found different combinations of sub- and super-Poissonian electron and phonon Fano factors depending on the asymmetry and on the strength of the electron-phonon coupling,

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- [1] G. Breitenback, S. Schiller, and J. Mlynek, *Nature* **387**, 471 (1997).
 - [2] H. J. Kimble, M. Dagenais, and L. Mandel, *Phys. Rev. Lett.* **39**, 691 (1977).
 - [3] Y. M. Blanter and M. Büttiker, *Phys. Rep.* **336**, 2 (2000).
 - [4] C. W. J. Beenakker and H. Schomerus, *Phys. Rev. Lett.* **93**, 096801 (2004).
 - [5] R. Sanchez, G. Platero, and T. Brandes, *Phys. Rev. Lett.* **98**, 146805 (2007).
 - [6] M. Blencowe, *Phys. Rep.* **395**, 159 (2004); A. N. Cleland, *Foundations of Nanomechanics*. (Springer, Berlin, 2003).
 - [7] H. Park *et al.*, *Nature (London)* **407**, 57 (2000).
 - [8] R. G. Knobel and A. N. Cleland, *Nature* **424**, 291 (2003).
 - [9] B. J. LeRoy *et al.*, *Nature* **432**, 371 (2004).
 - [10] L. Y. Gorelik, *et al.*, *Phys. Rev. Lett.* **80**, 4526 (1998); T. Novotný, A. Donarini, and A.-P. Jauho, *ibid.* **90**, 256801 (2003).
 - [11] J. Koch and F. von Oppen, *Phys. Rev. Lett.* **94**, 206804 (2005); J. Koch, F. von Oppen, and A. V. Andreev, *Phys. Rev. B* **74**, 205438 (2006).
 - [12] M. D. La Haye *et al.*, *Science* **304**, 74 (2004); A. Naik *et al.*, *Nature (London)* **443**, 193 (2006).
 - [13] A. D. Armour, M. P. Blencowe, and K. C. Schwab, *Phys. Rev. Lett.* **88**, 148301 (2002); A. Zazunov, D. Feinberg, and T. Martin, *ibid.* **97**, 196801 (2006).
 - [14] D. H. Santamore, A. C. Doherty, and M. C. Cross, *Phys. Rev. B* **70**, 144301 (2004); I. Martin and W. H. Zurek, *Phys. Rev. Lett.* **98**, 120401 (2007); K. Jacobs, P. Lougovski, and M. Blencowe, *ibid.* **98**, 147201 (2007).
 - [15] A. Mitra, I. Aleiner, and A. J. Millis, *Phys. Rev. B* **69**, 245302 (2004).
 - [16] T. Brandes and N. Lambert, *Phys. Rev. B* **67**, 125323 (2003).
 - [17] D. A. Rodrigues, J. Imbers, and A. D. Armour, *Phys. Rev. Lett.* **98**, 067204 (2007).
 - [18] A. N. Korotkov, *Phys. Rev. B* **49**, 10381 (1994).
 - [19] F. Haupt *et al.*, *Phys. Rev. B* **74**, 205328 (2006).
 - [20] A. Cottet, W. Belzig, and C. Bruder, *Phys. Rev. Lett.* **92**, 206801 (2004); F. Cavaliere *et al.*, *Phys. Rev. B* **71**, 235325 (2005).
 - [21] Yu. V. Nazarov and J. J. R. Struben, *Phys. Rev. B* **53**, 15466 (1996).
 - [22] D. F. Walls and P. Zoller, *Phys. Rev. Lett.* **47**, 709 (1981).